Interactive Digital Textbooks for Math Learning

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Abstract

Major structural changes in our modern society require a new national curriculum, based on generic skills necessary in the 21st century. As basic knowledge in Mathematics and Science is one of the most important components, it puts the question of developing some digital textbooks for learning Mathematics. What features should be met, what changes would occur in the work of a teacher, which models should be chosen in order to make Mathematics more accessible to all students - are the questions we try to answer in this paper. We would like to illustrate this by presenting an educational software for a Geometry lesson, "Discovering and Proving Circle Properties", for the seventh grade, developed by the author after completing the Professional Development Program “The teacher – Creator of Educational Software”, project financed by European Funds, Sectorial Operational Program Human Resources Development 2007-2013.

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1. Interactive Digital Textbooks for Math Learning – Need and Opportunity

Technological developments in pace exponentially along with the massive traffic of information, which minimises distances, physical or cultural, in a way unimaginable until recently, resulting in the globalisation of the society and changing the way of thinking and understanding the world of new generations. Under such circumstances, remodelling the educational paradigm becomes a necessity. Along with the classic (printed) textbooks and the traditional approaches to the educational process, new methods, based on recently developed technologies, are gaining ground.

The European educational policy reveals the need for rethinking the school curricula, based on generic competencies identified as being necessary in the 21st century, and the textbook is still considered the main instrument for the implementation of the curriculum through its attributed functions: “the transmission of knowledge, development of skills and competencies, consolidation of acquisitions (knowledge, abilities, skills), organisation and management of the process of learning, evaluating students’ acquisitions, support for integrating acquisitions, social and cultural education.” ([1]). Thanks to new technologies, there arises the opportunity of developing and using digital textbooks, able to offer a richer learning experience compared with traditional textbooks, which might lead to better results.

Pedagogues, psychologists and experts in educational design, teachers and professors, experts in e-learning and IT do studies that reveal how new technological environments for learning, from computers to tablets or smart-phones, can provide more efficient and interactive ways of learning, thus getting students to solve problems closer to the ones which might occur in real life. We have recently come to definitions of digital textbooks ([2]), as well as, prototypes of digital books, at present tested in our schools ([3]). On the other hand, the limits and the risks of inadequate digitisation of the learning content should not be ignored. ([4])
Beyond the general principles that underlie the development of digital textbooks, we have in mind the specificity of the chosen school discipline – it is not in the same way that we teach Mathematics or Literature – and the level of the study – the characteristics of students’ development level entail different approaches at the level of primary education, lower secondary, respectively upper secondary.

“Under the conditions in which information sources get multiplied, and the access to knowledge is now open to more and more people, the textbook (either digitised or classic support) must place emphasis on the use of knowledge, on building the approach towards investigating a specific domain and the ability to transfer knowledge and techniques of intellectual work from one area of knowledge to another. The focus on the student’s work becomes more and more important.” ([1]) In the case of Mathematics, digitisation supports these desiderata. By exploiting the facilities of new technologies, it is possible to create an optimal environment for using the method of solving problems, learning through practice and self-assessment. Thus, this digital textbook will acquire the role of a Practice Book, a “magic” one, which shall notify the student if the solution is correct, giving him an immediate feedback, which is a factor with important role in motivating and improving learning.

2. A Digital Mathematics Lesson – Lower Secondary Level

National and international assessments show that the overall level of the students’ preparedness in Mathematics and Science is not satisfactory. The advanced countries, in terms of technology, pay a lot of attention to the training of skills in Mathematics and Science – in USA, the Math Now Program aims at advanced Math programs for primary and secondary school, Japan develops “top science schools” for Mathematics, Science and Technology, through an innovative curriculum, for future scientists to ensure progress, Korea sets up classes for advanced studies that are offered only to those who have passed the compulsory topics, in Taiwan, there are classes for brilliant students in Mathematics and Science ([1]). Moreover, the educational policy of these countries emphasises the central idea to encourage students not only to assimilate scientific information, but also, to develop capacity to look at the world and to make it interact in a scientific way.

US K12 standards, set out through Science For All Americans, suggest that educational activities should focus on connections between Science and other disciplines, connections that occur naturally in the physical and biological world studied by scientists, but which are rarely shown in the traditional curriculum. In Romania, the first steps are taken in this direction only now by proposing the development of new curricula, including at the level of each school subject at least one trans-disciplinary or cross-disciplinary module.

The Maths lesson we want to present in this paper is addressed to the students from the VII\textsuperscript{th} or VIII\textsuperscript{th} grades, having as topic the main notions related to the\textsuperscript{th} Geometry of the Circle, the discovery and justification of geometric properties in a circle and how we can apply them in an interdisciplinary context. The lesson also contains a module dedicated to practice and assessment, decisive stage for building good competencies in Mathematics for all students.

2.1 Dynamic and interactive presentation of the notions

Taking into account the age of the target group, we stake on the ludicrous aspect of the lesson, with a role in motivating the students. Thus, this lesson will be accompanied by a character named Squary, coming from a strange world, in which the Circle is completely unknown. The student will have the opportunity to interact with the learning material. The images are dynamic, changing their appearance through interaction, revealing, one by one, the components of the geometric figures.
In the first part of the lesson, the student knows the basic concepts related to the geometry of the circle. The access to the nine components is non-linear, through the menu shown in Figure 1. The disks corresponding to the completed components have a different colour from those which have not been completely done yet.

The process of understanding the geometric notions is facilitated through the use of dynamic images: unlike the printed textbook, the digital textbook allows the presentation of the circle as the trajectory of a moving point, which is situated at a constant distance – the radius of the circle – from another fix point, called the centre of the circle.

The nine moments of the lesson, accessed via the menu shown in Figure 1, contain both definitions of geometric concepts, together with interactive images, which will highlight the key components and will provide the intuitive support, and exercises with immediate feedback, aiming to fix and verify the acquired notions, as well as the ability to use them.

For example, the image shown in Figure 2 presents the definition of the arc of a circle. The small arc $AB$ had changed its colour to red, after the user tapped one of its points; there was also displayed its name and the appropriate notation. When you have reached a certain point on the major arc $AB$, the latter will become red and will display its name, while the little arc will return to its original colour.

After defining the arcs of the circle, there are presented the relations between the measure of the central angle and the measure of the arcs and some exercises are proposed in order to apply the
formulas. The student will fill in the boxes with the calculated values, and the application will provide a feedback through the character *Squary* that jumps gleefully at every good answer; when the correct answer is introduced, the button for moving on to the next frame of the lesson will be activated.

In order to give the formula for calculating the length of the circle, an interactive simulation is used (*Figure 3*): a wire that surrounds the circle unrolls until it takes the form of a straight segment; with the help of the ruler, the students will measure the length of the wire, then they will be able to introduce the number, expressed in millimetres, in the appropriate box; in the same way, the students will measure the diameter of the circle, then will calculate the ratio of the two lengths and fill in the numbers. The application checks the introduced data, provides feedback and advances gradually as the tasks are performed correctly. The diameter which will be measured is constructed by simply tapping the centre of the circle. In the end, after finding out and stating the relationship between the length and the radius of a circle, an exercise is proposed.

### 2.2 The discovery and justification of the properties – from intuition to scientific rigour

Learning Mathematics in school is not limited to the formulation of rules or the application of algorithms. By learning Mathematics, we learn to do more complex reasoning; the role of Mathematics is recognised in training the qualities necessary for abstract, coherent and critical, properly articulated thinking. Thus, our teaching approach will rise from the intuitive level, based on observations, to the level of scientific reasoning and rigour.

*Figure 4*. Interactive simulation – we measure and calculate the value of PI

We will illustrate this statement by presenting the proof to one of the theorems included in the application (*Figure 4*). The demonstration of the theorem of the angle inscribed in a circle brings the student an interesting example of how solving a particular case of a problem can help to solve the general case.

In the left panel of the application, we can see the demonstration of the theorem when one side of the angle passes through the centre of the circle; by tapping the centre of the circle, the radius $OA$ is constructed, then we apply the property related to the exterior angle of a triangle and the theorem is rapidly justified for this particular case. The student will follow the reasoning and will complete the required data in the two boxes.

The central panel presents the demonstration of the theorem when the centre of the circle is situated inside the inscribed angle: by tapping the centre of the circle, we construct the diameter from the vertex of the angle, and the measure of the inscribed angle will be the sum of the two angles formed, which we can calculate like in the previous case.
Similarly, the right panel shows the demonstration of the theorem when the centre of the circle is located outside the angle: through a similar auxiliary construction, the angle is written as the difference between two angles similar to those in the left panel.

Then, the theorem will be applied for the angle inscribed in a semicircle. There is included another dynamic, interactive image: the students have the possibility to move the vertex of the angle anywhere on the semicircle, viewing that the measure of the angle remains constant and the arms of the angle are always perpendicular.

All these dynamic images, together with the tasks that students have to solve, will help them to be more involved, thus building a correct representation of the notions and relationships in geometry. Compared to static images in the printed textbook, the digital lesson makes the understanding of the notions more accessible to more students. The immediate feedback, given after solving each requirement, plays an important role in improving the learning process.

2.3 Interdisciplinarity and transfer of knowledge

The second main section of the lesson proposes nine problems to be solved, with practical and interdisciplinary nature. The access to these problems can be non-linear, from the second menu of the application. As an example, which we show in Figure 5, the way in which the student is led in the solving of the problem of the determination of the diameter of a ball’s shadow, if the distance from the light source to the wall and the distance from the ball to the light source are known. The reasoning is built step by step, the student will fill in successively the requested numbers by applying the theorems learned – the Pythagorean theorem and the similarity of triangles –, finally getting the desired result.

The six boxes that are to be completed for solving the problem show the path to be followed by the students to construct their reasoning. In order to complete the first box, the students must realize that the segment is actually the radius of the circle, whose diameter is given. We must note that although this requirement is trivial, a significant number of students may have difficulties in solving them! It is about the phenomenon of functional illiteracy: although the students can read, memorise and reproduce certain statements, in fact they do not understand and cannot apply what they read – a phenomenon which can be reduced only by solving such exercises every day.
The degree of difficulty in completing the 6 boxes increases gradually: for the second box the students must realize that the length can be calculated as the sum of the distances from the light source to the ball and the radius of the ball; for the third box the students have to apply the Pythagorean theorem in a right triangle, for the fourth box they will complete the relationship of proportionality of similar triangles’ sides, thus being able to complete the last but one box. The final answer to the problem will be introduced in the last box, using the same relationship between the radius and the diameter of a circle. The feedback is provided after completing each box, when the user clicks on the Enter or Verify button.

2.4 The role of practice in achieving mathematical competencies

Basic skills in Mathematics and Science are among the key competencies identified as necessary for our graduates of the compulsory education. However, the results of the national assessment and baccalaureate exams show that a large percentage of our graduates were unable to meet the minimal requirements. The situation could be improved if every student had the possibility to practise independently, in their own pace, solving the types of questions proposed to these exams. Experience shows that the vast majority of students can improve their performance in Mathematics through practice, if they notice that their results are improving. The digital textbook will be able to provide the assisting function in the process of learning, with the possibility to practise until the students are able to use the notions and work techniques correctly.

For the lesson Circle and its properties we created a set of nine items with auto generating data. We want to exemplify this with the following item, with over a quarter of a million variants (namely 257 544 variants), which randomly auto generates at each run of the test. Here is a variant of the item: the points $A_1, A_2, \ldots, A_{72}$ divide a circle into 72 equal parts. Find the measure of the angle $A_5A_{15}A_{40}$. The variant was generated using the following pattern: The points $A_1, A_2, \ldots, A_n$ divide a circle into $n$ equal parts. Find the measure of the angle $A_iA_jA_l$. The generator randomly picks a divisor $k$ of 60, then considers $n=6k$ and chooses for $i$ a random value between 1 and $k$, for $j$ a random value between $k+1$ and $3k$, and for $l$ a random value between $3k+1$ and $6k$. For each set, the application calculates the correct answer, in order to provide feedback to the student. The student will enter the answer, then will operate the verification button. If you have entered the correct answer, you can move on to the next item, otherwise you can try again. The application also contains a button which allows the display of hints for each problem.
3. Conclusions

The main priorities of the current teaching methodology are the need for personalised learning approach and the emphasis on learning through cooperation. Personalised approach in learning Mathematics at lower secondary school level can be accomplished either through a significant reduction in the number of students in a class (10-12) or through the development of digital textbooks, able to assist the students in the learning process. At lower secondary level, the students are only at the stage of mathematical initiation, skills are formed by working individually, at their own pace, until the proper knowledge and techniques are gained. If the current and the final evaluations will be conducted in accordance with the items included in the digital textbooks, the students will have all the necessary motivation to practise, having the above described self-assessment tool.

It is necessary for the teacher to perfectly know all the learning contents in order to support the students with appropriate explanations. The teacher’s encouragement, along with improved scores at each run of the tests, will win the students for more practice with greater confidence in this exciting game of mind: Mathematics – a game through which we form higher cognitive skills, analytical and conceptual thinking, skills related to personal effectiveness, self-control, self-confidence, flexibility, competencies oriented towards development and action, concern for order, searching for information and development of intuition – competencies that are needed in today’s world, which calls for a continuous adaptation through innovation.

On the other hand, the cooperative learning of Mathematics, in the case of lower secondary level, begins with the knowledge and practice of the rules of the dialogue: the students will learn to listen carefully to their classmates’ views and reasoning and to clearly express their own point of view. Consequently, working with chalk and blackboard in Mathematics is not and will not be outdated soon. For the success of such an approach, we need teachers to transpose mathematical knowledge to a language accessible to the students, with flexibility in approaches and observing the principle of intrinsic mathematical rigour.

The transition from stating these principles to their application in school, day after day, requires the involvement and cooperation of all actors of the educational system, starting with the decision makers, which should act fairly to make a real change – not just in words.

4. References